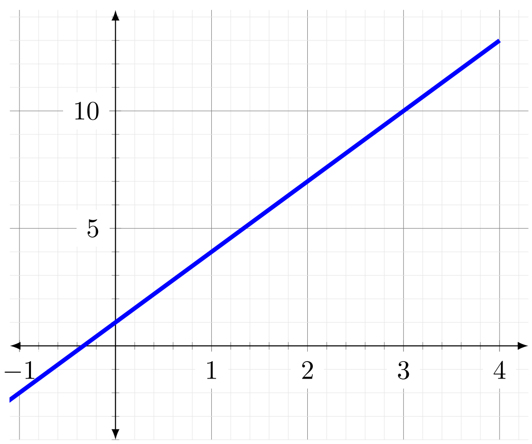
LECTURE NOTES 2-4: THE PRECISE DEFINITION OF THE LIMIT (DAY 2)

REVIEW: THE PRECISE DEFINITION OF THE LIMIT:

We say $\lim_{x \to a} f(x) = L,$ if for *every* number $\epsilon > 0$, there exists a number $\delta > 0$ such that

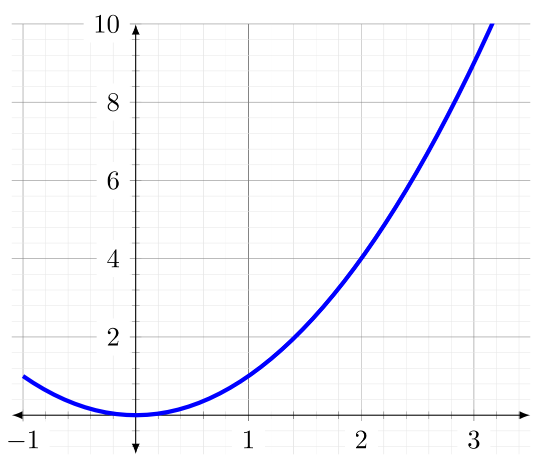
$$\text{if } 0 < |x - a| < \delta, \text{ then } |f(x) - L| < \epsilon.$$

Recall that we used this definition to show that $\lim_{x \to 2} 3x + 1 = 7$.

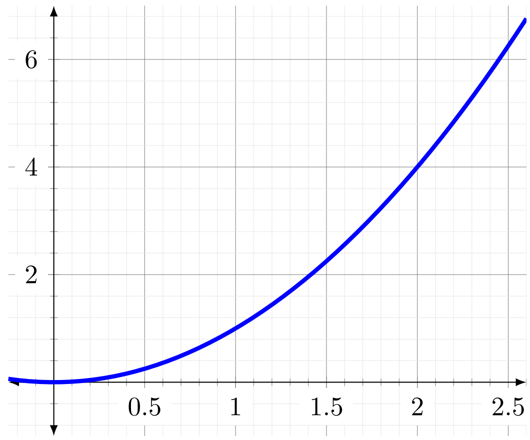


PRACTICE PROBLEMS:

1. Let $f(x) = x^2$, graphed below.



(a) Find a number δ such that if $|x - 2| < \delta$, then $|x^2 - 4| < 1$. Use the graph to show that your answer is correct.



(b) Find a number δ such that if $|x-2| < \delta$, then $|x^2-4| < \frac{1}{5}$. Use the graph to show that your answer is correct.

(c) How is finding δ different if the function, $f(x)$, is not linear?

2. A machinist is required to manufacture a metal cube with a volume of $8000\ cm^3$.

   (a) What side length produces such a cube?

   (b) If the machinist is allowed an error tolerance of $\pm 10\ cm^3$ in the volume of the cube, how close to the ideal side length in part (a) must the machinist control the radius?

   (c) In terms of the ϵ-δ definition of $\lim_{x\to a} f(x) = L$, what is:

      i. x?
      ii. $f(x)$?
      iii. a?
      iv. L?
      v. ϵ?
      vi. δ?